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Saltatory de Sitter string vacua

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Abstract: We extend a recent scenario of Kachru, Kallosh, Linde and Trivedi to fix the string moduli fields by using a combination of fluxes and non-perturbative superpotentials, leading to de Sitter vacua. In our scenario the non-perturbative superpotential is taken to be either, the racetrack scenario or the $\mathcal{N} = 1^*$ superpotential for an SU(N) theory, originally computed by Dorey and recently rederived using the techniques of Dijkgraaf-Vafa. The fact that this superpotential includes the full instanton contribution gives rise to the existence of a large number of minima, increasing with N. In the absence of supersymmetry breaking these correspond to supersymmetric anti de Sitter vacua. The introduction of antibranes lifts the minima to a chain of (non-supersymmetric) de Sitter minima with the value of the cosmological constant decreasing with increasing compactification scale. Surprisingly a similar picture occurs for the simpler system of the racetrack scenario. The relative semiclassical stability of these vacua is studied. Possible cosmological implications of these potentials are also discussed.

Keywords: D-branes, Superstring Vacua.

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1. Introduction

Supersymmetry breaking and moduli fixing have been the main obstacles for string theory to make contact with low energy physics. These questions are also essential for the study of any possible cosmological implications of the theory. Over the years there have been several proposals to solve these problems. Supersymmetry breaking mechanisms include the effect of non-perturbative field theoretical effects such as gaugino condensation and, more recently, the explicit breaking due to the presence of antibranes or brane intersections

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in low scale string models. The remnant potential for the moduli fields in these cases is such that the global minimum is either anti de Sitter space with a very large (negative) vacuum energy or the potential is of the runaway type, towards infinite extra dimension and/or zero string coupling. The problem is then not how to break supersymmetry but actually what to do, after breaking it, with the remaining potential for the moduli.

Recently there has been interesting progress towards the solution of the continuum vacuum degeneracy problem in string theory. The introduction of fluxes of RR or NS-NS forms [1]–[5] has proven to be very efficient to fix many of the moduli fields, including the dilaton. However in typical orientifold (or F-theory) compactifications, the overall Kähler modulus cannot be fixed by this effect [2, 4].

An interesting proposal to also fix this modulus, due to Kachru, Kallosh, Linde and Trivedi (KKLT) [6], combines the fluxes with non-perturbative superpotentials that have been discussed in string theory in the past, consisting of a single exponential of the corresponding modulus. The modulus is then stabilised for a supersymmetric anti de Sitter point. The further inclusion of anti D3 branes breaks supersymmetry explicitly and can lift the minimum of the potential to a de Sitter minimum with varying value of the cosmological constant, depending on the different parameters of the theory. Even though this is a very simple set-up, involving much fine tuning and the simplest non-perturbative superpotential, it represents a concrete step forward towards fixing the moduli after supersymmetry breaking, with a potentially realistic value of the vacuum energy.

In a different direction, progress has also been made in the understanding of non-perturbative effects in supersymmetric field theories. In particular, several techniques have been developed that allow the computation of the exact non-perturbative superpotentials. In some simple cases they have been derived in closed form encoding the infinite sum of exponentials of the inverse gauge coupling expected from the full instanton and fractional instanton effects.

In this note, we slightly extend the KKLT proposal by considering more general non-perturbative superpotentials than the single exponential considered in KKLT. In particular we consider the non-perturbative superpotential for a supersymmetric theory for which the exact superpotential has been computed explicitly, including the infinite instanton sum. This is the so-called $\mathcal{N} = 1^*$ theory [7, 8].

The structure of the potentials generated in this way is such that there are many AdS supersymmetric minima in the absence of the antibranes and many de Sitter minima in their presence, with intermediate configurations having both dS and AdS vacua, all non-supersymmetric. The minima are such that the value of the vacuum energy decreases with increasing value of the compactification scale. This rich vacuum structure may have interesting physical implications and can serve as a concrete nontrivial example illustrating the possible 'landscape' of string theory [9]. It is also similar to the staircase potentials proposed by Abbott [10] and to the models presented in [11, 12, 13] exhibiting a dynamical relaxation of the cosmological constant. Furthermore, there is enough freedom to fine tune one of the minima to have a cosmological constant as small as required. The presence of several de Sitter minima could also lead to interesting realisations of inflation.

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We also consider simpler cases in which the superpotential is a finite sum of exponentials as it appears in the much studied racetrack scenarios [14]. These are the simplest extensions of the mechanism of KKLT and, in the cases that lead to several minima, they serve as simple examples to study the transition between different vacua.

This article is organised as follows: After briefly reviewing the effect of fluxes to fix the moduli in the next section, we start considering in section 3, the general scalar potential for the Kähler modulus field for arbitrary superpotential. In section 4 we first consider the case of superpotentials with two exponentials which can give rise to one or two different minima. Section 5 discusses the potential for the $\mathcal{N}=1^*$ theory with its rich vacuum structure with and without breaking supersymmetry. Section 6 is dedicated to the stability analysis of the different minima. We recover in particular the results of KKLT about the life time of the dS minimum with smallest positive value of the cosmological constant which is much larger than the age of the universe but smaller than the Poincaré recurrence time. We also discuss the relative probability for tunneling between different minima.

2. Fluxes and moduli fixing

Type-IIB strings have RR and NS-NS antisymmetric 3-form field strengths, H_3 and F_3 respectively, that can have a (quantised) flux on 3-cycles of the compactification manifold.

$$\begin{aligned} \frac{1}{4\pi^2\alpha'}\int_A F_3 &= M\,, \\ \frac{1}{4\pi^2\alpha'}\int_B H_3 &= -K\,, \end{aligned} \tag{2.1}$$

where K and M arbitrary integers and A and B different 3-cycles of the Calabi-Yau manifold.

The inclusion of fluxes of RR and/or NS-NS forms in the compact space allows for the existence of warped metrics that can be computed in regions close to a conifold singularity of the Calabi-Yau manifold, with a warp factor that is exponentially suppressed, depending on the fluxes, as:

$$a_0 \sim e^{-2\pi K/3g_s M}\,. \tag{2.2}$$

Therefore fluxes can naturally generate a large hierarchy. Here g_s is the string coupling constant.

Fluxes have also proven very efficient for fixing many of the string moduli, including the axion-dilaton field of type-IIB theory $S = e^{\phi} + i\hat{a}$. A very general analysis of orientifold models of type IIB, or its equivalent realisation in terms of F-theory, has been done by Kachru and collaborators [4, 5]. In the F theory approach, the geometrical picture corresponds to an elliptically fibered four-fold Calabi-Yau space Z with base space $\mathcal{M}$ and the elliptic fiber corresponding to the axion-dilaton field S.

The consistency condition in terms of tadpole cancellation implies a relationship between the charges of D-branes, O-planes and fluxes that can be written as follows:

$$N_{\rm D3} - N_{\bar{\rm D}3} + N_{\rm flux} = \frac{\chi(Z)}{24}\,, \tag{2.3}$$

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where the left hand side counts the number of D3 branes and antibranes as well as the flux contribution to the RR charge:

$$N_{\text{flux}} = \frac{1}{2\kappa_{10}^2 T_3} \int_M H_3 \wedge F_3 \,. \tag{2.4}$$

The r.h.s. of (2.3) refers to the Euler number of the four-fold manifold Z or in terms of orientifolds of type IIB, to the contribution of the D3-brane charge due to orientifold planes and D7-branes. Here κ_{10} refers to the string scale in 10D and T_3 to the tension of the D3 branes.

The fluxes generate a superpotential on the low-energy four-dimensional effective action of the Gukov-Vafa-Witten form [3]:

$$W = \int_M G_3 \wedge \Omega \,, \tag{2.5}$$

where $G_3 = F_3 - iSH_3$ with S the dilaton field and Ω is the unique $(3,0)$ form of the corresponding Calabi-Yau space.

In the simplest models there will be one single Kähler structure modulus defining the overall size of the Calabi-Yau space which we denote by $T = r^4 + ib$ where r is the scale of the extra dimensions and b an axion field coming from the RR 4-form ($T = i\rho$ in the conventions of [4, 6]). The relevance of this modulus is that it is the one that cannot be fixed by the fluxes. Its Kähler potential is of the no-scale form, that is:

$$K = \tilde{K}(\varphi_i, \varphi_i^*) - 3\log\left(T + T^*\right) , \tag{2.6}$$

with $\tilde{K}$ the Kähler potential for all the other fields φ_i except for T. This implies that the supersymmetric scalar potential takes the form

$$V_{\text{SUSY}} = e^K \left(K^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} \right) , \tag{2.7}$$

with $K^{i\bar{j}}$ the inverse of the Kähler metric $K_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$ and $D_i W = \partial_i W + W \partial_i K$ the Kähler covariant derivative. The T dependence of the Kähler potential is such that the contribution of T to the scalar potential cancels precisely the term $-3e^K|W|^2$ of the standard supergravity potential, this is the special property of no-scale models [15]. Since this potential is positive definite, the minimum lies at zero, with all the fields except for T fixed from the conditions $D_i W = 0$. This minimum is supersymmetric if $D_T W = W = 0$ and not supersymmetric otherwise.

Since the superpotential does not depend on T, we can see in this way that the fluxes can fix all moduli but T. In order to fix T KKLT procede as follows:

1. Choose a vacuum in which supersymmetry would be broken by the T field, such that $W = W_0 \neq 0$.

2. Consider a non-perturbative superpotential generated by euclidean D3-brane or by gaugino condensation due to a non-abelian sector of wrapped N D7-branes. For which

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the gauge coupling is $\frac{8\pi^2}{g^2_{\rm YM}} = 2\pi \frac{r^4}{g_s} = 2\pi\,{\rm Re}\,T$. Which induces a superpotential of the form $W_{np} = Be^{-2\pi T/N}$. Combining the two sources of superpotentials $W_0 + W_{np}$, they find an effective scalar potential with a non-trivial minimum at finite T and the standard runaway behaviour towards infinity, as usual. The non-trivial minimum corresponds to negative cosmological constant giving rise to a supersymmetric AdS vacuum.

3. In order to obtain de Sitter vacua, KKLT, consider the effect of including anti D3 branes, still satisfying the condition (2.3). This has the net effect of adding an extra (non-supersymmetric) term to the scalar potential of the form:

$$V = V_{\rm SUSY} + \frac{D}{({\rm Re}\,T)^3} \tag{2.8}$$

with the constant $D = 2a_0^4 T_3/g_s^4$ paramterizing the lack of supersymmetry of the potential. Here a_0 is the warp factor at the location of the anti D3 branes and T_3 the antibrane tension. The net effect of this is that for suitable values of D the original AdS minimum gets lifted to a dS one with broken supersymmetry. See figure 1.

Here we will modify the KKLT scenario in two ways. First, regarding the original fluxes, we can consider the supersymmetric configuration where $W_0 = 0$ with the form G_3 being of the $(2,1)$ type. That is we may include a non-vanishing W_0 part in the superpotential but it will not be necessary. Second, regarding the non-perturbative superpotential, we explore the simplest $\mathcal{N} = 1$ supersymmetric model for which the full non-perturbative superpotential has been computed, including the contribution from all (infinite) instantons. This is the so-called $\mathcal{N} = 1^*$ model, constructed from mass deformation of $\mathcal{N} = 4$ super Yang-Mills.

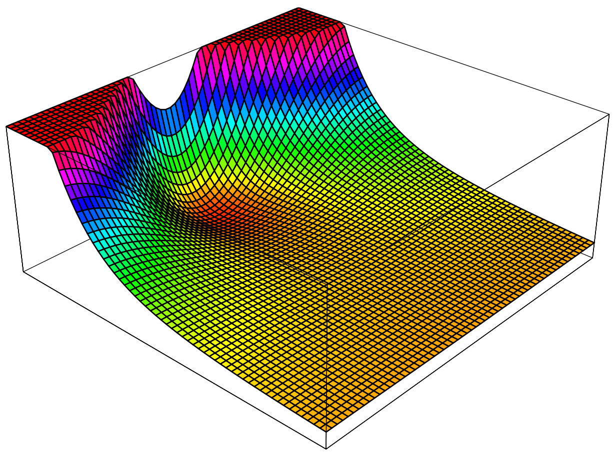

Figure 1: The scalar potential considered in [6] with one single de Sitter minimum.

Our first minor modification allows to start with an explicit supersymmetric model, before considering the low-energy non-perturbative effects. This avoids the need to fine tune the value of W_0 in looking for non-trivial minima. Our second modification allows exploring the possibility of an exact instanton calculation, instead of a single instanton calculation as it is usually considered. We will see that this exact superpotential will have a constant piece (similar to W_0) and an infinite sum of exponential terms, allowing for a very rich vacuum structure. For completeness, we also considered simpler superpotentials including a sum of two exponentials as in the racetrack scenario.

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3. The general scalar potential

3.1 The supersymmetric potential

The standard $\mathcal{N} = 1$ supergravity formula for the potential in Planck units reads

$$V_{\mathrm{SUSY}} = e^K \left(\sum_{i,j} K^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2 \right), \tag{3.1}$$

where i, j runs over all moduli fields. As we already mentioned, $K^{i\bar{j}} = \partial_i \partial_{\bar{j}} K$ where K is the corresponding Kähler potential and $D_i W = \partial_i W + (\partial_i K) W$. In our case we are working with a model having only one Kähler modulus, (that is, $h^{1,1}(M) = 1$) as we will be focusing on the T field. All other fields are assumed to have been fixed by the fluxes just as in [6] so the superpotential W will depend on the superfield T.

Then our purpose is to study the scalar potential $V(T)$ which also depends on the Kähler potential. We take the weak coupling result in 4-dimensional string models, namely

$$K = -3 \log(T + T^*), \tag{3.2}$$

and neglect possible perturbative and non-perturbative corrections. For simplicity of notation we will write the field T in terms of its real and imaginary parts:

$$T \equiv X + iY. \tag{3.3}$$

Using (3.1) and (3.2) the scalar potential turns out to be

$$V_{\mathrm{SUSY}} = \frac{1}{8X^3} \left\{ \frac{1}{3} |2XW' - 3W|^2 - 3|W|^2 \right\}, \tag{3.4}$$

where by $'$ we understand derivatives with respect to the field T. To compute the supersymmetric minima of the scalar potential we need to calculate the derivative of (3.4) that is given by

$$V' = \frac{\partial V}{\partial T} = \frac{(2XW'' - 2W')(2XW' - 3W)^* - 2(W')^*(2XW' - 3W)}{24X^3}, \tag{3.5}$$

Notice then that there can be two types of extrema. The supersymmetric extrema appear when

$$2XW' - 3W = 0. \tag{3.6}$$

In this case the value of the potential at the extremum is clearly negative definite leading to an anti de Sitter vacuum. These extrema are minima provided that:

$$|XW'' - W'| > |W'|. \tag{3.7}$$

The nonsupersymmetric extrema occur at

$$(XW'' - W') = (W')^* e^{2i\gamma}, \tag{3.8}$$

where $\gamma = \arg(2XW' - 3W)$.

In all cases considered here, our analysis shows that the condition (3.8) is never fulfilled at the minima. Then all minima are supersymmetric and lead to negative cosmological constants.

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3.2 Supersymmetry breaking

Following the lines of [6], to uplift the anti de Sitter vacua to de Sitter vacua we will introduce in our model a $\overline{\mathrm{D3}}$. This will break the supersymmetry of the system. The introduction of the antibrane does not introduce extra translational moduli as its position is fixed by the fluxes [16], so it just contributes to the energy density of the system. This contribution is given by [16]

$$\delta V = \frac{D}{X^3}\,, \tag{3.9}$$

where the coefficient D is a function of the tension of the brane T_3 and of the warp factor a_0 and has the form $D = 2\frac{a_0^4 T_3}{g_s^4}$, where g_s denotes the string coupling. The coefficient D depends on the fluxes through the dependence of the warp factor a_0 on them and it is therefore quantised, although the range of values of the fluxes can be such that for practical purposes it may be considered as an almost continuum variation [6, 27].

If we add this supersymmetry breaking term to the scalar potential (3.4) we have now that the expression for the scalar potential is given by

$$V = V_{\mathrm{SUSY}} + \delta V\,. \tag{3.10}$$

The effect of the supersymmetry breaking term depends on the range of values of the parameter D. If D is very small, the potential will not change substantially and the minima remain anti de Sitter. For a critical value of D one of the minima will move up to zero vacuum energy and then to de Sitter space. Continuing increasing D, more minima become de Sitter until all of them are de Sitter. If D is larger than another critical value the nonsupersymmetric term starts dominating the potential and starts eliminating the different extrema to make the potential runaway with X. The range of values of D will depend very much on the form of W and we will illustrate the behaviour in the examples below.

4. Superpotentials with two exponentials

Our main interest in this paper is to consider non trivial superpotentials including all instanton contributions in terms of an infinite sum of exponentials. For simplicity we will first discuss the simpler case of superpotentials with just two exponentials. We then consider the following superpotential

$$W = W_0 + A\,e^{-aT} + B\,e^{-bT}\,. \tag{4.1}$$

4.1 The racetrack scenario

This superpotential has an interest by itself since it includes the standard racetrack scenario (when $W_0 = 0$), very much discussed in order to fix the dilaton field at weak coupling. The origin of the so-called racetrack scenario is the condensation of gauginos of a product of gauge groups. For an $\mathrm{SU}(N)\times\mathrm{SU}(M)$ group we would have $a = 2\pi/N$ and $b = 2\pi/M$. For large values of N and M, with M close to N the minimum is guaranteed to lie in the large field region. A similar result will happen for the field T taken to be the gauge coupling at a

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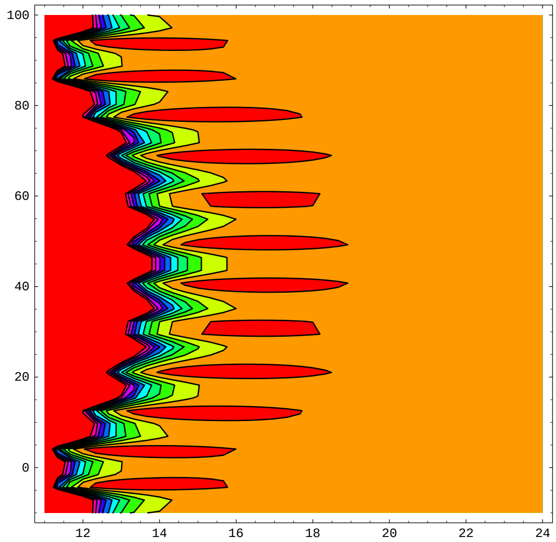

Figure 2: Contour plot for a racetrack type potential with several minima.

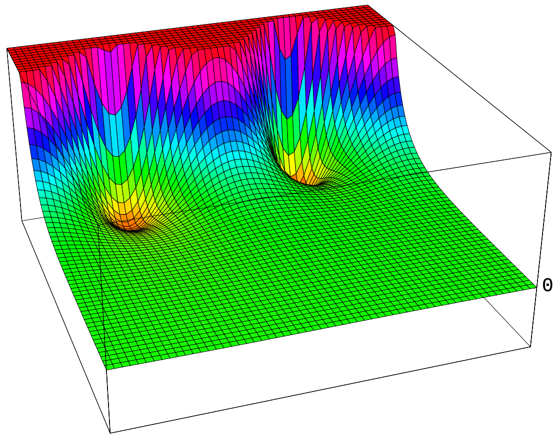

Figure 3: Graph of the scalar potential with $a = 0.1$, $A = 1$, $B = 5\cdot 10^4$ and $W_0 = -10^{-3}$.

product gauge group coming from the D7 sector. Turning on W_0 it is possible to find more than one minimum that, following the KKLT method, would then lead to many (order 10) de Sitter vacua depending on the values of the parameters W_0, N, M, A, B. In figure 2 we present a contour plot of the potential illustrating this behaviour with $W_0 \sim -10^{-3}$ and all other constants in the range $1, 10$.

4.2 A finite instanton sum

The potential takes a manageable form if we assume $b = 2a$ which allows more than one minima and we will follow mostly for illustration purposes. In this case using (3.1), (3.2) and (4.1) the scalar potential turns out to be

$$V_{\text{SUSY}} = \frac{ae^{-4aX}}{6X^2}\Big[6B\,e^{2aX}W_0\cos(2aY) + 6B^2 + 3A^2\,e^{2aX} + 4aB^2X + \\ + aA^2X\,e^{2aX} + A\,e^{aX}(3\,e^{2aX}W_0 + B(9+4aX))\cos(aY)\Big]. \quad (4.2)$$

Notice that this potential is periodic in Y with period $2\pi/a$, and is also invariant under the reflexion $Y \to -Y$.

From (4.2) it is easy to see that if W_0 is negative ($W_0 < 0$) the minima of the potential will be located at $Y = Y^{(n)} = \pi n/a$, $n = 0, \pm 1, \pm 2, \ldots$ For each value of $Y^{(n)}$ we will find a minimum at $X = X^{(n)}$. Since the potential (4.2) is periodic in Y with period $2\pi/a$ we will just consider the cases $n = 0$ and $n = 1$, being the rest of the cases copies of these two. We will denote these minima as $(X^{(\pm)}, Y^{(\pm)})$, where $+$ will denote the case $n = 0$ and $-$ will denote the case $n = 1$. In figure 3 we show an explicit example of the potential (4.2).

At the minima of the potential we have that

$$\partial_X V\,|_{Y=Y^{(\pm)}} = 0 \quad \Longrightarrow \quad W_0 = -\frac{e^{-2aX^{(\pm)}}}{3}\left(\pm A\,e^{aX^{(\pm)}}(3 + 2aX^{(\pm)}) + B(3 + 4aX^{(\pm)})\right). \quad (4.3)$$

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Using this we can get the values of the potential at the minima. Those values are given by

$$V_0^{(\pm)} = -\frac{a^2}{6X^{(\pm)}} \, (\pm A \, e^{-aX^{(\pm)}} + 2B \, e^{-2aX^{(\pm)}})^2 \,. \tag{4.4}$$

Note that from (4.3) we find that $X^{(+)} > X^{(-)}$ and then from (4.4) $V_0^{(+)} > V_0^{(-)}$. Also note from (4.4) that we will always have two non-degenerate minima with null or negative value of the potential, that is, we will have either Minkowski or AdS vacua.

On the other hand, if W_0 is positive ($W_0 > 0$), the points located at $Y = Y^{(n)} = \pi n/a$, $n = 0, \pm 1, \pm 2, \ldots$ are no longer minima, but maxima. In this case we find that the scalar potential will have two degenerate minima in every $2\pi/a$ period of Y. Those minima fulfill $Y^{(1)} + Y^{(2)} = 2\pi/a$ and $X^{(1)} = X^{(2)}$ (the reason for that is the invariance of (4.2) under $Y \to -Y$). This means that we should need more exponential terms in (4.2) in order to have at least two non-degenerate minima. Therefore we will concentrate in the case $W_0 < 0$, as is the simplest case of that type. The analysis of the case $W_0 > 0$ is analogous to the case considered in [6], where they analyse the case of a scalar potential with just one minimum.

Now, we will study what is the effect of breaking supersymmetry in this model. The introduction of the supersymmetry breaking term (3.9) acts in the following way: when $D/(X^{(\pm)})^3 \ll V_0^{(\pm)}$ the potential is almost not affected, and we still have two AdS minima. If we increase the value of D such that $D/(X^{(\pm)})^3 \sim V_0^{(\pm)}$, the value of the scalar potential at the minima increases with D, so the minima will eventually become positive (dS minima). For larger values of D the minima become saddle points and then disappear.

Also notice that the supersymmetry breaking term (3.9) does not involve Y, therefore the new potential will also be periodic in Y with the same period as before, and its minima will also be located at $Y = Y^{(n)} = \pi n/a$, $n = 0, \pm 1, \pm 2, \ldots$ Again, for each value of $Y^{(n)}$ we will find a minimum in X but now at different values than in the supesymmetric case. We will denote these values as $\hat{X}^{(n)}$. For each of those values we will find again two different values of the potential. It is interesting to note that the position of the minima in the non-supersymmetric case ($X = \hat{X}^{(\pm)}, Y = Y^{(\pm)}$) does not vary significantly from the position of the minima in the supersymmetric case ($X = X^{(\pm)}, Y = Y^{(\pm)}$).

Therefore, for a given value of D such that $D/(X^{(\pm)})^3 \sim V_0^{(\pm)}$ the non-susy scalar potential will have two different minima that, depending on the value of the constants, can be either positive or negative. If we want to describe the present stage of the acceleration of the universe within this framework, we would like both minima to be dS minima, such that $V_0^{(+)} > V_0^{(-)} \sim 10^{-120}$ in Planck units. This can always be done in this model for example by fine-tuning the value of D. In figure 4 we show an explicit example where the two AdS minima that appear in the supersymmetric case shown in figure 3 become two dS minima in the non-supersymmetric case.

5. $\mathcal{N} = 1^*$ theory

In this section, we would like to study more general superpotentials were the full instanton sum has been computed, being the simplest such case the $\mathcal{N} = 1^*$ theory. The $\mathcal{N} = 1^*$

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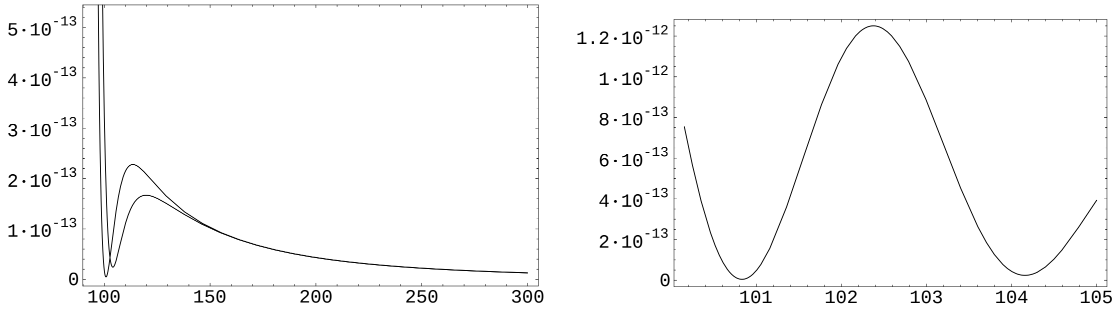

(a) Cross section of the scalar potential at $Y = 0$ and $Y = \pi/a$.

(b) Cross section of the scalar potential along a line that joins the two minima.

Figure 4: Example of a configuration with $a = 0.1$, $A = 1$, $B = 5 \cdot 10^4$, $W_0 = -10^{-3}$, and $D = 3.5 \cdot 10^{-7}$. In this case we find a configuration with two de Sitter minima.

theory is mass deformed $\mathcal{N} = 4$ super Yang-Mills in which the $\mathcal{N} = 1$ chiral multiplets inside the $\mathcal{N} = 4$ Yang-Mills multiplet are given a mass. At the moment we do not have a concrete example where this theory appears at low energies on the D7-branes. We may think of ways that it could arise, for instance since the D-branes break one half of the supersymmetries, they tend to have a $\mathcal{N} = 4$ gauge theory inside. Having a model that breaks supersymmetry to $\mathcal{N} = 1$ outside the brane would naturally induce masses to the chiral multiplets. However here we will take the $\mathcal{N} = 1^*$ superpotential only as an illustrative example of what we can expect from full instanton contributions to the superpotential.

5.1 Background

Let us first review what the $\mathcal{N} = 1^*$ theory is, starting from SU(N), $\mathcal{N} = 4$ super Yang-Mills. The SU(N), $\mathcal{N} = 4$ super Yang-Mills theory can be written in terms of $\mathcal{N} = 1$ superfields as a gauge theory with three massless chiral superfields Φ_i in the adjoint of SU(N) with superpotential

$$W = \epsilon_{ijk} \operatorname{Tr} \Phi_i [\Phi_j, \Phi_k] . \tag{5.1}$$

A deformation of this theory by adding mass terms to these superfields

$$\Delta W = m_i \operatorname{Tr} \Phi_i^2 , \tag{5.2}$$

with all $m_i \neq 0$ breaks supersymmetry to $\mathcal{N} = 1$. This is the $\mathcal{N} = 1^*$ theory. There are further deformations of the original $\mathcal{N} = 4$ theory that may be considered [18].

The classical vacua of this theory can be found by solving $\partial W_T / \partial \Phi_i = 0$ for $W_T = W + \Delta W$, which leads to

$$[\Phi_i, \Phi_j] = \epsilon_{ijk} m_k \Phi_k \tag{5.3}$$

and therefore the solutions correspond to N-dimensional representations of the SU(2) algebra, giving rise to the different phases of the theory.

The massive (Higgs and confining) phases of this theory are well understood. They are labelled by a triplet of integers (p, q, k) with $pq = N$ and $0 < k < q$. These phases

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interpolate from the full confining phase $q = N$ to the full Higgs phase $p = N$. The exact superpotential for $\mathcal{N} = 1^*$ was derived by using instanton techniques for the theory compactified on a circle. The compactification to three dimensions is a computational trick and it turns out that the superpotential is independent of the compactification radius [7]. After integrating out the gauge fields, the superpotential depends only on the (complex) gauge coupling τ and takes the form:[1]

$$W_{p,q,k}(\tau) = E_2(\tau) - \frac{p}{q} E_2\left(\frac{p}{q}\tau + \frac{k}{q}\right). \tag{5.4}$$

Here and later in this section by E_n we will denote the Eisenstein modular functions.

The modular properties of the superpotential show that under a $\mathrm{SL}(2,\mathbb{Z})$ transformation

$$\tau \to \frac{a\tau + b}{c\tau + d} \tag{5.5}$$

the effective theory in a given phase is not invariant but it exchanges different phases by changing the values of p, q, k. For instance $\tau \to \tau + 1$ requires $(p, q, k) \to (p, q, k + p)$ and $\tau \to -1/\tau$ implies $(p, q, k) \to (\alpha, \frac{N}{\alpha}, k')$ with $\alpha = \gcd(q, k)$ and k' having a complicated dependence on p, q, k with $k' = 0$ if $k = 0$, indicating the exchange of Higgs and confining phases under this transformation.

It is interesting to note that the validity of this exact non-perturbative result has been checked using string theory techniques [8] and more recently from Dijkgraaf-Vafa techniques [17, 18], making this expression very robust.

5.2 The scalar potential

Following [19] we will now promote the parameter τ to a full $\mathcal{N} = 1$ superfield that we identify with the modulus field[2] $T = -i\tau$. Then the superpotential (5.4) can be written in terms of T as

$$W_{p,q,k}(T) = E_2(T) - \frac{p}{q} E_2\left(\frac{p}{q}T - i\frac{k}{q}\right), \tag{5.6}$$

The superpotential $W_{p,q,k}(T)$ transforms as a modular form of weight 2 once the values of p, q, k are transformed accordingly. As we already mentioned, the net effect of a $\mathrm{SL}(2,\mathbb{Z})$ transformation is that it changes one massive phase to a different phase.

For concreteness we will concentrate in the confining phase $p = 1$, $q = N$, for which it is enough to set $k = 0$, since $k \neq 0$ can be reached by a translation $T \to T - ik$. In

[1]An overall factor of order $m^3 = \frac{N^3}{24} m_1 m_2 m_3$ appears multiplying the superpotential, where m_i are the masses of the chiral superfields of the $\mathcal{N} = 4$ theory. This will scale the scalar potential by a particular scale, which we take as unity for simplicity, but need to keep it in mind when combining with the non-supersymmetric case and to take care of actual number estimates regarding the value of the cosmological constant. For consistency we have to take the scale set by the m_i's to be hierarchically smaller than the string scale. It is not yet clear if these small mass parameters will be achieved in explicit models.

[2]In reference [19] the corresponding field was the dilaton S rather than T then the difference between the potential calculated there and the supersymmetric potential we consider here is only in the factor of 3 in the Kähler potential. The main difference appears when we consider the nonsupersymmetric case.

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this case the scalar potential will be given by (3.4) where now the superpotential and its derivative take the form

$$W(T) = E_2(T) - \frac{1}{N} E_2\left(\frac{T}{N}\right), \tag{5.7}$$

$$W'(T) = \frac{\pi}{6}\left\{(E_4(T) - E_2(T)^2) - \frac{1}{N^2}\left(E_4\left(\frac{T}{N}\right) - E_2\left(\frac{T}{N}\right)^2\right)\right\}. \tag{5.8}$$

We will mostly work with the $E_n(T)$ expansions in terms of the variable $q = e^{-2\pi T}$ that are given by

$$E_2 = 1 - 24\sum_{n=1}^{\infty} \sigma_1(n) q^n\,, \qquad E_4 = 1 + 240\sum_{n=1}^{\infty} \sigma_3(n) q^n\,, \tag{5.9}$$

where $\sigma_p(n)$ is the sum of the p^{th} powers of all divisors of n. E_4 is a modular form of weight 4 so $E_4(1/T) = T^4 E_4(T)$, while E_2 fails to be a modular form of weight two since

$$E_2\left(\frac{1}{T}\right) = -T^2 E_2(T) + \frac{6T}{\pi}. \tag{5.10}$$

Since there is already a constant term in the expansion of the superpotential we will perform most of our analysis considering $W_0 = 0$ which means no supersymmetry breaking by the fluxes themselves. Contrary to the case with one single exponential, we will find many minima without needing to tune the value of W_0 to obtain large compactification volume naturally once we break supersymmetry. We will see later how W_0 affects our results.

5.2.1 Numerical analysis

In order to perform reliable computations with the Eisenstein series we use the 'weak coupling' or large radius expansion of $W(T)$ in eq. (5.6) only when $2\pi X > N$. For other ranges we can use the property (5.10). For instance, for $1 < 2\pi X < N$ we can transform the $E_2(T/N)$ term to obtain

$$W(T) = E_2(T) + \frac{N}{T^2} E_2\left(\frac{N}{T}\right) - \frac{6}{\pi T}. \tag{5.11}$$

Similarly, when $2\pi X < 1$[3] we can transform both terms in $W(T)$ to obtain

$$W(T) = -\frac{1}{T^2} E_2(T) + \frac{N}{T^2} E_2\left(\frac{N}{T}\right). \tag{5.12}$$

N	2	3	4
X	1.46	1.70	1.82
$V_{\min}$	$-1.73 \cdot 10^{-2}$	$-1.60 \cdot 10^{-2}$	$-1.32 \cdot 10^{-2}$

Table 1: Minima of the scalar potential.

For $N \leq 4$ we have that the minima appear when $2\pi X > N$, so that one has to use the expression (5.7) for the superpotential. One finds the supersymmetric minima are at $Y = N/2$ and X (when $N \leq 4$) at the values given in table 1.

For $N \geq 5$ the flat direction at $Y = N/2$ turns into a saddle point and pairs of supersymmetric minima T_{i1}, T_{i2} on either side in the Y direction appear, such that $Y_{i1} + Y_{i2} = N$ and $X_{i1} = X_{i2}$. Examples of these are given in table 2.

[3] We have to keep in mind that in these regimes there may be substantial corrections to the Kähler potential which are not under control.

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N	5	6	7	8	9
X	1.48	1.65	1.77	1.85	1.88
Y	2.24	2.40	2.63	2.88	3.13
$V_{\rm min}$	$-1.35 \cdot 10^{-2}$	$-1.37 \cdot 10^{-2}$	$-1.27 \cdot 10^{-2}$	$-1.16 \cdot 10^{-2}$	$-1.07 \cdot 10^{-2}$

Table 2: Minima of the scalar potential for several cases.

$N = 30$			$N = 40$			$N = 50$		
X	Y	$V_{\rm min}$	X	Y	$V_{\rm min}$	X	Y	$V_{\rm min}$
1.20	6.44	$-1.12 \cdot 10^{-2}$	1.28	6.95	$-5.72 \cdot 10^{-3}$	1.10	7.52	$-3.43 \cdot 10^{-3}$
1.32	11.71	$-1.34 \cdot 10^{-2}$	1.44	8.73	$-1.68 \cdot 10^{-2}$	1.15	14.36	$-8.58 \cdot 10^{-3}$
1.57	8.40	$-1.70 \cdot 10^{-2}$	1.60	15.31	$-1.54 \cdot 10^{-2}$	1.36	8.92	$-1.36 \cdot 10^{-2}$
1.65	12.15	$-1.16 \cdot 10^{-2}$	1.76	11.32	$-1.46 \cdot 10^{-2}$	1.59	10.98	$-1.70 \cdot 10^{-2}$
2.08	5.69	$-5.21 \cdot 10^{-3}$	1.84	16.77	$-1.06 \cdot 10^{-2}$	1.76	18.97	$-1.39 \cdot 10^{-2}$
			2.17	6.56	$-4.19 \cdot 10^{-3}$	1.82	14.24	$-1.26 \cdot 10^{-2}$
						1.92	21.30	$-9.10 \cdot 10^{-3}$
						2.24	7.32	$-3.51 \cdot 10^{-3}$

Table 3: Minima of the scalar potential for $N = 30, 40, 50$.

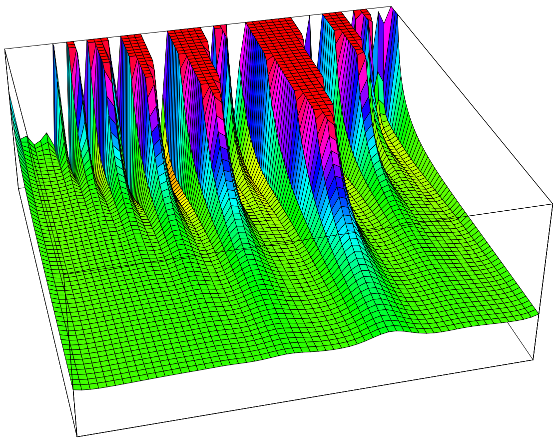

Figure 5: Graph for the scalar potential with $N = 100$.

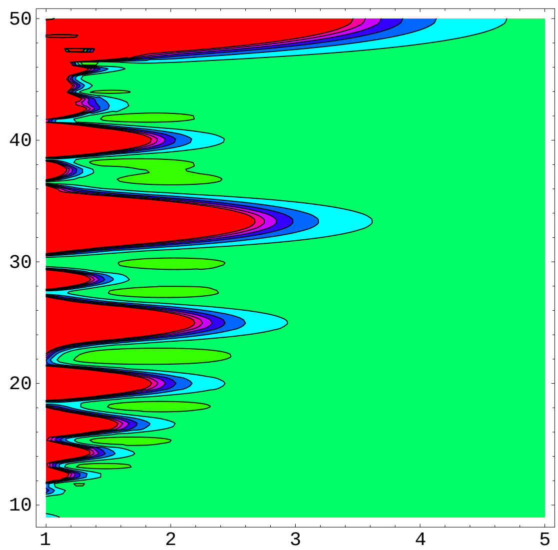

Figure 6: Contour graph for the scalar potential with $N = 100$.

For $N \geq 10$ we have that the minima begin to appear when $2\pi\, X < N$, so that one has to use the expression (5.11) for the superpotential to compute the scalar potential (3.4). This needs to be done in order to have a convergent series expansion in (5.9). Using this we find a lot of supersymmetric minima in the scalar potential, noting that the number of minima increases with N. We list some of the results table 3.

Also we present in figure 5 a 3D plot of the potential for the case $N = 100$ illustrating the rich structure of the potential with many supersymmetric AdS minima. In figure 6 we present a contour plot showing the location of the minima and how the landscape changes in field space.

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N	Number of	Min_G			Min_X		
	Minima	X	Y	$V_{\min}$	X	Y	$V_{\min}$
10	2	1.28	3.59	$-1.11 \cdot 10^{-2}$	1.85	3.32	$-1.01 \cdot 10^{-2}$
20	3	1.76	7.84	$-1.43 \cdot 10^{-2}$	1.96	4.65	$-6.92 \cdot 10^{-3}$
30	5	1.57	8.40	$-1.70 \cdot 10^{-2}$	2.08	5.69	$-5.21 \cdot 10^{-3}$
40	6	1.44	8.73	$-1.69 \cdot 10^{-2}$	1.28	6.95	$-5.73 \cdot 10^{-3}$
50	7	1.59	10.98	$-1.70 \cdot 10^{-2}$	1.10	7.52	$-3.46 \cdot 10^{-3}$
60	10	1.45	10.78	$-1.70 \cdot 10^{-2}$	1.20	7.62	$-1.62 \cdot 10^{-3}$
70	10	1.53	12.62	$-1.76 \cdot 10^{-2}$	2.32	8.61	$-2.67 \cdot 10^{-3}$
80	10	1.62	14.45	$-1.69 \cdot 10^{-2}$	1.13	8.85	$-1.07 \cdot 10^{-3}$
90	10	1.49	13.75	$-1.78 \cdot 10^{-2}$	2.38	9.73	$-2.16 \cdot 10^{-3}$
100	10	1.55	15.28	$-1.76 \cdot 10^{-2}$	2.41	10.25	$-1.97 \cdot 10^{-3}$
200	15	1.52	20.98	$-1.76 \cdot 10^{-2}$	2.56	14.37	$-1.06 \cdot 10^{-3}$
300	19	1.60	26.00	$-1.74 \cdot 10^{-2}$	2.70	17.50	$-7.26 \cdot 10^{-4}$
400	20	1.49	29.59	$-1.80 \cdot 10^{-2}$	2.68	20.20	$-5.53 \cdot 10^{-4}$
500	25	1.47	32.20	$-1.77 \cdot 10^{-2}$	2.70	22.60	$-4.46 \cdot 10^{-4}$
600	25	1.50	36.31	$-1.79 \cdot 10^{-2}$	2.73	24.70	$-3.74 \cdot 10^{-4}$
700	27	1.47	37.80	$-1.75 \cdot 10^{-2}$	2.75	26.60	$-3.22 \cdot 10^{-4}$
800	31	1.62	45.70	$-1.72 \cdot 10^{-2}$	2.77	28.43	$-2.83 \cdot 10^{-4}$
900	34	1.56	46.10	$-1.76 \cdot 10^{-2}$	2.78	30.10	$-2.52 \cdot 10^{-4}$
1000	38	1.48	46.50	$-1.72 \cdot 10^{-2}$	2.78	31.80	$-2.28 \cdot 10^{-4}$

Table 4: Number of minima for different values of N in the supersymmetric case.

To illustrate the fact that we find an increasing number of minima when we increase N, and also other interesting facts, we present in table 4 some other results of the numerical analysis. Note that by Min_G in table 4 we denote the global minimum of the potential and by Min_X we denote the minimum with a largest (finite) value of X.

From the information shown in table 4 we may extract some general remarks regarding the minima. First, the number of minima grows quite fast when N grows. Also the potential at the minimum with larger value of X increases with N. Furthermore, even though we do not have a general proof, for all the cases we have analysed, all the vacua of this theory happen to be supersymmetric.

Finally, although it is not necessary in our case, following [6] we have studied the effect of turning on W_0. Notice that with $W_0 = 0$ we have many minima but all of them correspond to not too large values of the compactification scale X. We have found that tuning W_0 has the effect of allowing minima with very large values of X improving the validity of the field theoretical analysis of the potential. Otherwise the general structure of the potential remains the same.

5.2.2 Analytical considerations

The scalar potential (3.4) appears to be, in general terms, too complicated to do an analytical study of its minima for a superpotential such as the $\mathcal{N} = 1^*$ one. In fact, introducing the superpotential (5.11) into the condition (3.8) that is used to find the supersymmetric minima we arrive to non linear equations that cannot be solved analytically.

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In any case, some interesting features appear when we study the behaviour of the potential in the limit $(2\pi X)\,N \gg X^2+Y^2$. In that limit the superpotential W can be well approximated by the function

$$W \sim 1 + \frac{N}{T^2} - \frac{6}{\pi T}\,. \tag{5.13}$$

This approximation is derived from (5.11) just by keeping the constant terms in the E_2 expansions and neglecting the exponential terms.

Using the approximate superpotential in (5.13), we find that the condition for a supersymmetric minimum, i.e., $2XW' - 3W = 0$, reduces to a cubic equation in X which can be solved analytically. This cubic equation is given by:

$$\left(\frac{288}{\pi} + 2N\pi\right) X - 168X^2 + 24\pi X^3 = 18N\,, \tag{5.14}$$

with Y given in terms of X by the following relation:

$$Y = \sqrt{N + \frac{X(3\pi X - 16)}{\pi}}\,. \tag{5.15}$$

The full form of the solutions for X and Y is not very enlightening, but they allow an expansion in powers of $1/N$ that happens to be more useful.[4]

$$X_{\rm min} = \frac{9}{\pi} - \frac{3240}{\pi^3}\frac{1}{N} + \frac{5015520}{\pi^5}\frac{1}{N^2} + \cdots \tag{5.16}$$

$$Y_{\rm min} = \sqrt{N}\left(1 + \frac{99}{2\pi^2}\frac{1}{N} - \frac{502281}{8\pi^4}\frac{1}{N^2} + \cdots\right) \tag{5.17}$$

It is clear from these expressions that when N is large the solution for X will tend asymptotically to $9/\pi$, and Y will tend to $\sqrt{N}$. It is also possible to compute the value of the potential in this minimum (also as an expansion in power series of N) substituting the approximate expression for the superpotential W given in (5.13) into (3.4). Its expansion in powers of $1/N$ is given by

$$V_{\rm min} = -\frac{2\pi}{27}\frac{1}{N} + \frac{50}{3\pi}\frac{1}{N^2} + \cdots\,. \tag{5.18}$$

In fact, this minimum is the minimum with the largest finite value of X. Also we have found that for $N \geq 50$ these results (5.14), (5.15) and (5.18) agree well with those obtained in the numerical analysis (see table 4). This minimum is in fact the minimum that appears to have the largest value of the potential for large values of N. This will play an important role in the next section, where we explore the minima of the potential when the supersymmetry is broken.

[4] The equation (5.14) has of course three solutions for X, but only one of the three solutions of the cubic equation gives a positive $X_{\rm min}$ in the range of validity of the approximation.

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5.3 Supersymmetry breaking

In this subsection we will study the changes produced in the structure of the vacua when we introduce in the scalar potential the supersymmetry breaking term (3.9). We will see that with the introduction of such a term it is possible to lift the vacua from anti de Sitter vacua to de Sitter vacua, for some range of the parameter D.[5]

5.3.1 Analytical considerations

As in the previous case, the scalar potential (3.10) appears to be, in general terms, too complicated to do an analytical description of its minima. In fact, introducing the superpotential (5.11) into the scalar potential (3.10) and minimizing it to find the minima leads to non linear equations that cannot be solved analytically.

In any case, similarly to the previous cases, some interesting features appear when one studies the behaviour of the potential in the limit $2\pi\, X\, N \gg (X)^2 + (Y)^2$. As in the previous case, in that limit the superpotential W can be well approximated by the function (5.13).

Using the approximate superpotential in (5.13), we find that the condition for minimum reduces to two polynomial equations in X , Y. Those equations cannot be solved in general but for N large it is easy to find that Y is given in terms of X by the following relation:

$$Y = \sqrt{\frac{5N\pi X + 6X^2(\pi X - 7)}{6(\pi X - 1)}}\,. \tag{5.19}$$

Using this relation (5.19), we find a complicated polynomial equation on just X. This equation can be simplified for large N to a cubic polynomial on X^2, and therefore can be solved analytically. In fact, what we found is that only two of the solutions are positive and then lead to two real positive solutions for X. Nevertheless just one of these two solutions is a minimum, the other one corresponding to a saddle point. The solution found is proportional to $\sqrt{N}$, with the proportionality constant given in terms of the supersymmetry breaking parameter D. That is, the solutions for the minimum are given by $X_{\rm min} = f(D)\, N^{1/2}$.

The exact form of the function $f(D)$ is a complicated expression, but in any case it is possible to extract some useful features from it. In fact, it is easy to see that $f(D)$ is an increasing function on D, until it reaches an upper bound D_M. For values $D > D_M$ the values of the function $f(D)$ are no longer real, this meaning that the potential would have no minima. Therefore, in the limit of validity of the analytical analysis, there is a bound for the supersymmetry breaking parameter D if we want the potential to have minima. This bound is given by

$$D < \frac{1}{360}(27 + 7\sqrt{21}) \equiv D_M\,. \tag{5.20}$$

[5] Recall that the $\mathcal{N} = 1^*$ superpotential has a mass scale m^3. This was irrelevant for the discussion of the supersymmetric case since it could be rescaled out of the potential, affecting only its absolute value at the minima. We have to keep this in mind when considering the range of variation of the parameters W_0 and D which will carry now an extra scale determined by m.

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Therefore, for values of the supersymmetry breaking parameter D larger than this bound we will not find any minima of the scalar potential.

When the bound in D is saturated we find that

$$X^M_{\rm min} = \sqrt{\frac{5}{12(-4+\sqrt{21})}}\, N^{1/2}\,, \tag{5.21}$$

These values for X and D allow an expansion in powers of $1/N$ of the potential in the minimum. We find that the value of the potential is positive and is given by

$$V^M_{\rm min} = 0.0406\frac{1}{N^{3/2}} + 0.0386\frac{1}{N^2} + \cdots \tag{5.22}$$

Also from the form of $f(D)$ it is clear that one can fine-tune the supersymmetry breaking parameter D so that the value of the scalar potential in the minimum $X_{\rm min}$ approaches arbitrarily to zero. In fact one can compute the value of D such that $V_{\rm min} = 0$. The full expression is not very useful, but it allows an expansion in power series in $1/N$, which is given by

$$D_0 = \frac{3}{20} - \frac{9}{20\pi}\sqrt{\frac{3}{5N}} + \frac{99}{1600\pi^2}\frac{1}{N} + \cdots\,, \tag{5.23}$$

and also the values of $X_{\rm min}$ and $Y_{\rm min}$ are

$$X^0_{\rm min} = \sqrt{\frac{5}{12}N} + \frac{17}{8\pi} - \frac{99}{64\pi^2}\sqrt{\frac{3}{5N}} + \cdots \tag{5.24}$$

$$Y^0_{\rm min} = \sqrt{\frac{5}{4}N} - \frac{1}{8\pi\sqrt{3}} - \frac{3463}{192\pi^2}\sqrt{\frac{1}{5N}} + \cdots \tag{5.25}$$

where the value of $Y^0_{\rm min}$ is obtained from (5.19). Also remember that $V^0_{\rm min} = 0$.

In fact, we have found that for $N \geq 50$ these results agree well with those obtained in the numerical analysis, as we will show in the following subsection.

5.3.2 Numerical results

In this subsection we present the results found in the numerical analysis of the scalar potential in the non-supersymmetric case. We have computed numerically the minima of the potential for $D = D_0$ for several values of N, such as $N = 50$, $N = 100$, $N = 500$ and $N = 1000$. The results obtained from the analysis are shown in table 5.

As a matter of comparison with the supersymmetric case, we show in figure 7 the scalar potential for $N = 100$ with supersymmetry broken.

Finally we show in table 6 a similar information as the one shown in table 4 in the last subsection, where we write the number of minima varying with N. In table 6 we denote by ${\rm Min}_{X,Y}$ the minima with largest value of X, Y. We can notice that the number of minima still increases with N but slower than in the supersymmetric case. An argument for this is that the non-supersymmetric term δV tends to smooth out the potential as it moves the minima up, so some of the minima eventually disappear. Another implication of this term is that the minima tend to have larger value of the compactification scale, large enough to create a hierarchy as compared to the string scale, as desired phenomenologicaly.

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$N=50$			$N=100$		
X	Y	$V_{\min}$	X	Y	$V_{\min}$
2.63	14.24	$1.71 \cdot 10^{-3}$	2.95	22.24	$1.23 \cdot 10^{-3}$
4.46	20.48	$3.36 \cdot 10^{-4}$	3.77	37.26	$5.36 \cdot 10^{-4}$
5.46	7.94	0	4.62	44.20	$1.35 \cdot 10^{-4}$
			7.35	11.24	0
$N=500$			$N=1000$		
X	Y	$V_{\min}$	X	Y	$V_{\min}$
2.88	47.61	$1.88 \cdot 10^{-3}$	3.02	68.96	$1.74 \cdot 10^{-3}$
3.40	52.65	$1.18 \cdot 10^{-3}$	5.89	192.46	$9.23 \cdot 10^{-5}$
5.43	118.30	$1.31 \cdot 10^{-4}$	5.89	207.54	$9.23 \cdot 10^{-5}$
5.46	131.72	$1.30 \cdot 10^{-4}$	6.69	240.77	$4.18 \cdot 10^{-5}$
6.31	158.00	$4.62 \cdot 10^{-5}$	6.69	259.23	$4.18 \cdot 10^{-5}$
6.30	175.33	$4.62 \cdot 10^{-5}$	8.24	321.20	$1.38 \cdot 10^{-5}$
8.49	237.24	$8.59 \cdot 10^{-6}$	8.24	345.46	$1.38 \cdot 10^{-5}$
15.33	25.11	0	11.46	482.05	$2.42 \cdot 10^{-6}$
			21.34	35.51	0

Table 5: Minima of the scalar potential for several non-supersymmetric cases.

Furthermore, unlike the supersymmetric case, the minima appear to be ordered in X, where by ordered we mean that the value of the potential decreases as X increases. The one with largest compactification scale has a smallest cosmological constant.

In figure 8 we illustrate the effect of the non-supersymmetric term in the potential. The value of the potential at the minima is presented for several values of the parameter D. For $D = 0$ we have the supersymmetric case with more minima and not ordered, the increasing of the parameter D will reduce the number of minima, increase the value of the compactification scale at the minima and order them from larger to smaller values of the cosmological constant as the radius increases.

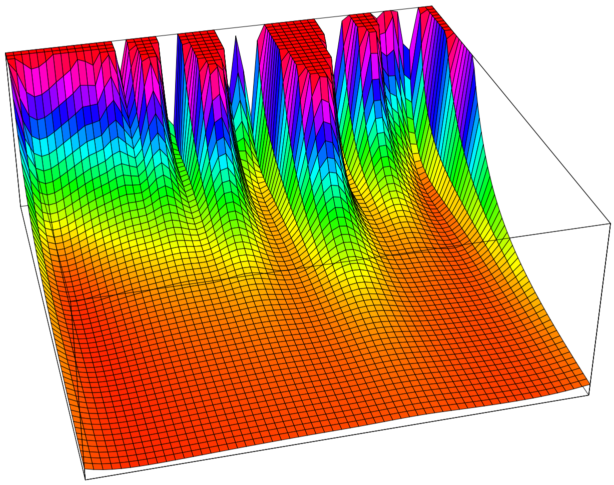

Figure 7: Graph for the non-supersymmetric scalar potential with $N = 100$.

Turning on a nonvanishing value of W_0 tends to change some of these conclusions. The minimum with the largest value of X can be lifted to positive vacuum energy while some of the other ones remain anti de Sitter. Besides this, the number of minima stays of the same order.

We can see that, in general, increasing the value of N substantially increases the number of vacua and the value of X at the minima. Notice that N can be quite large

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N	Number of Minima	Min$_X$			Min$_Y$		
		X	Y	$V_{\rm min}$	X	Y	$V_{\rm min}$
10	2	3.98	3.90	0	3.98	3.90	0
20	2	3.82	5.00	0	3.82	5.00	0
30	2	4.42	6.10	0	2.88	12.00	$7.69 \cdot 10^{-4}$
40	2	4.99	7.10	0	3.49	16.30	$4.60 \cdot 10^{-4}$
50	3	5.46	7.94	0	4.46	20.48	$3.36 \cdot 10^{-5}$
60	3	5.89	8.70	0	4.06	25.50	$3.09 \cdot 10^{-4}$
70	4	6.29	9.40	0	4.17	30.10	$2.42 \cdot 10^{-4}$
80	4	6.63	10.00	0	4.28	34.80	$1.92 \cdot 10^{-4}$
90	4	7.04	10.70	0	4.55	39.40	$1.60 \cdot 10^{-4}$
100	4	7.35	11.24	0	4.62	44.20	$1.35 \cdot 10^{-4}$
200	5	10.02	16.00	0	5.94	91.80	$4.30 \cdot 10^{-5}$
300	5	12.03	19.40	0	6.61	107.50	$1.13 \cdot 10^{-4}$
400	6	13.76	22.40	0	7.73	124.50	$7.52 \cdot 10^{-5}$
500	6	15.40	25.10	0	8.54	156.70	$5.48 \cdot 10^{-5}$
600	8	16.70	27.50	0	9.29	211.00	$4.21 \cdot 10^{-5}$
700	8	17.97	29.70	0	10.09	245.30	$8.03 \cdot 10^{-5}$
800	9	19.20	32.80	0	10.81	253.80	$2.79 \cdot 10^{-5}$
900	9	20.30	33.70	0	11.10	432.90	$2.35 \cdot 10^{-5}$
1000	9	21.34	35.51	0	11.46	482.05	$2.42 \cdot 10^{-6}$

Table 6: Minima for different values of N in a non-supersymmetric case with $D = D_0$.

in string theory and we may attempt extracting results varying N over many orders of magnitude. However, there are limiations on the numerical analysis if N is too large, since many more terms in the series have to be considered and the accuracy of the results would not be guranteed. Also computer and woman time to perform the analysis would increase substantially. Notice also that the number of minima we present here is actually a lower bound since, again, the accuracy of the numerical calculations could have missed some of them. In particular, minima which would be very close to each other could be missed.

6. Stability of the vacua

Having a situation with many nonsupersymmetric vacua with positive cosmological constant implies that the relative stability of this vacua should be considered. In particular if our own universe may be described by one of them, it is important to know the lifetime of each of the vacua against decay to any of the ones with smaller value of the cosmological constant and compare it with the age of the universe.

In the general case with many vacua the stability question is very much model and vacuum dependent. Given any particular minimum of the potential there is a nonvanishing probability towards tunneling to any of the other minima. The lifetime will depend on the height of the potential and the side of the local maxima or saddle points that separate

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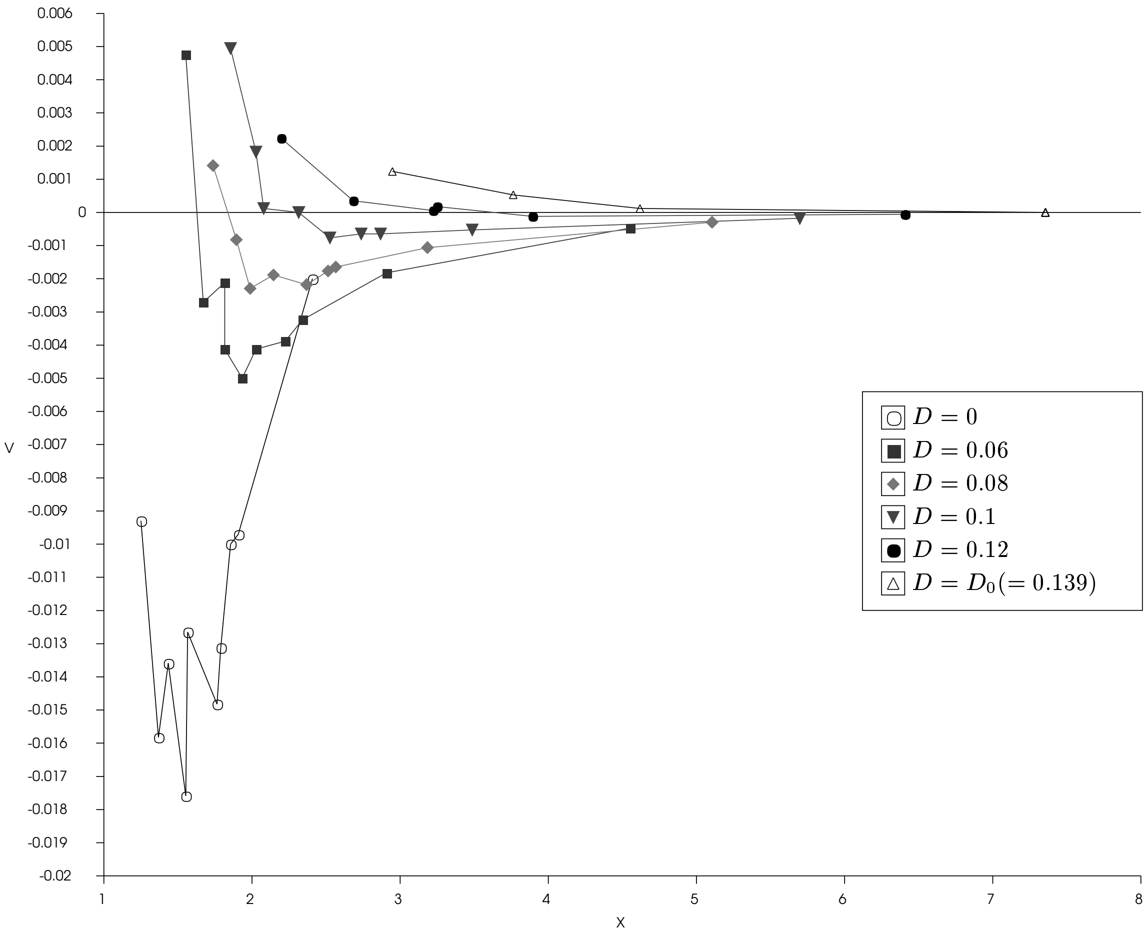

Figure 8: Potential versus X for $N = 100$ and different values of D.

two different minima. The general analysis is therefore very complicated. We will limit here to the well controlled case of two notrivial minima that we found in the case with two exponentials. This may serve as an illustration of the techniques used in the general case.

We know that the effective scalar potential has the standard runaway behaviour towards infinity in the radial modulus X. The existence of the runaway vacuum at ∞ in field space with zero energy is a common feature of all string theories [20]. This means that none of the two minima that appear in the scalar potential (see figures 3, 4) will be stable, as they are unstable, not only to decay from one minimum to the other minimum, but also to decay to the runaway minimum at $X \to \infty$, by means of either quantum tunneling or thermal excitation over a barrier [21, 22].

As we already mentioned, if we want to describe the present stage of the acceleration of the universe using a model of the kind presented here, we would like the last de Sitter minimum to be so that $V \sim 10^{-120}$ in Planck units. This can always be done in these models just by adjusting the value of the different parameters of the theory. In the case presented in section 3, we can achieve this by fine-tuning W_0 and D, while in the $\mathcal{N} = 1^*$ such a value for the cosmological constant can be obtained by slightly modifying the value of $D = D_0$ (for which the value of the potential is zero in the last minimum) by a small amount.

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Doing so one can always get the desired value for V in the last de Sitter minimum. We will restrict here to the analysis of the case considered in section 3, so we will have two minima with different values of the scalar potential $V_0^{(+)}$, $V_0^{(-)}$ such that $V_0^{(+)} > V_0^{(-)} \sim 10^{-120}$. Therefore we will have a model with several decay possibilities. In fact from the minimum at $V_0^{(+)}$ it is possible to decay either to the minimum at $V_0^{(-)}$ or to the minimum at $X \to \infty$, while from the minimum at $V_0^{(-)}$ it is just possible to decay to the minimum at $X \to \infty$. We will also comment some features of the $\mathcal{N} = 1^*$ case. In order to analyse this issue we will review several features of tunneling theories taking into account gravitational effects, following the original work of Coleman and De Luccia [21].

6.1 Vacuum decay

Let us consider a theory of a scalar field φ with a potential $V(\varphi)$ which has two local minima at φ_1, φ_2 with $V(\varphi_1) > V(\varphi_2)$. Both of the minima are stable classically but the vacuum state at $\varphi = \varphi_1$ (that is, the false vacuum) is unstable against quantum tunneling and will finally decay into the true vacuum state at $\varphi = \varphi_2$, this vacuum decay proceeding through the materialisation of a bubble of true vacuum within the false vacuum phase. The tunneling action is given by

$$S(\varphi) = \int d^4x\sqrt{g}\left(-\frac{1}{2}R + \frac{1}{2}(\partial\varphi)^2 + V(\varphi)\right), \tag{6.1}$$

with a tunneling probability between two vacua, per unit time and unit volume, given by

$$P(\varphi) \approx e^{-B} = e^{-S(\varphi)+S(\varphi_1)}. \tag{6.2}$$

Here φ is a solution of the equations of motion, which is usually referred to as "the bounce", and $S(\varphi_1)$ is the euclidean action in the initial configuration $\varphi = \varphi_1$. In the limit of small energy density between the two vacua Coleman and De Luccia showed that the coefficient B can be calculated in a closed form. Although in the general case it is usually very difficult to find analytical solutions for the Coleman-De Luccia instantons and calculate the probability of decay through quantum tunneling, the computation can be simplified within the range of validity of the thin-wall approximation. This approximation is valid when the thickness of the transition region between the true and the false vacuum is small compared with the radius of the bubble. In Minkowski space, the condition $V_{\rm min} \ll V_{\rm max}$ usually means that the thin-wall approximation is applicable, so the false vacuum state will decay through the materialisation of a bubble of true vacuum within the false vacuum phase, which is a quantum tunneling effect (by $V_{\rm max}$ we denote the high of the de Sitter maximum that separates two minima). That condition is usually fulfilled in the model considered here, and we will explicitly check later that in those cases we are always within the limits of the thin-wall approximation.

Now let us consider the case in which we have a vacuum with positive cosmological constant (a dS vacuum) and a vacuum with null cosmological constant (a Minkowski vacuum).[6] For those cases Coleman and De Luccia found that the decay will be produced

[6]This is always our case unless in the decay from $V_0^{(+)}$ to $V_0^{(-)}$, that are both de Sitter vacua. Nevertheless, as $V_0^{(+)} \gg V_0^{(-)} \sim 10^{-120}$, it is a good approximation to consider that the decay is also from dS to Minkowski.

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through a nucleation of a bubble of radius[7] $\bar{\rho} = \frac{12 S_1}{4\epsilon + 3 S_1^2}$, where ϵ denotes the energy density difference between the two vacua $\epsilon = V(\varphi_1) - V(\varphi_2)$. As the final vacuum has null cosmological constant $V(\varphi_2) = 0$, then $\epsilon = V(\varphi_1)$. Also S_1 denotes the tension of the bubble wall, that is given by

$$S_1 = \int_{\varphi_1}^{\varphi_2} d\varphi \, \sqrt{2V(\varphi)} \,, \tag{6.3}$$

The coefficient B is then given by

$$B = \frac{24\pi/\epsilon}{(1 + 4\epsilon/3S_1^2)^2} \,. \tag{6.4}$$

The thin-wall approximation is justified in this context if $\bar{\rho}$ and $\sqrt{3/\epsilon}$ are large compared with the range of variation of the scalar field φ, that is, $\bar{\rho}, \sqrt{3/\epsilon} \gg \Delta\varphi$.

6.2 Comparing tunneling probabilities

In this subsection we will compute the probabilities of all possible tunneling trajectories between the different vacua of the potential (3.10), (4.2). Nevertheless before doing so we need to make a remark: In our case we have a potential that depends on two scalar fields. When two or more fields are involved obtaining the bounce φ, and therefore computing tunneling probabilities, becomes a much more difficult task (see for example [23]), except in trivial cases.

An example of a trivial case is to consider the tunneling from any of the minima to the runaway minimum at $X \to \infty$. In those cases the lines $Y = 0, \pi/a$ are always minima of the potential in the Y direction, so we can consider a one-dimensional potential $V^{(\pm,\infty)} = V(X, Y = 0, \pi/a)$ and analyse the tunneling probability in the standard way (we would obtain one-dimensional graphs like the ones shown in figure 4*b*). Also note that the scalar field φ is defined in such a way that the kinetic term in the action (6.1) is canonical. In our analysis, the complex scalar field T has a kinetic term that is given by the derivatives of the Kähler potential (3.2), and is given by $\frac{3}{4(X)^2}(\partial X \partial X + \partial Y \partial Y)$. If we consider that Y is fixed then the kinetic term coming from the Kähler potential reads $\frac{3}{4}(\partial \ln X)^2$, so the canonical field would be of the form $\varphi^{(\pm,\infty)} = \sqrt{\frac{3}{2}} \ln X$.

On the other hand, the case of computing the tunneling probability from the minimum $V_0^{(+)}$ to the minimum $V_0^{(-)}$ is more subtle, as we cannot trivially reduce it to the one-dimensional case in the same way as the previous case. It is however possible to find a lower bound for the tunneling probability by replacing the multi-field potential by a suitably chosen single field potential, as any chosen trajectory will have larger tension (6.3) than the minimum one. As we are interested in comparing probabilities such a bound will be enough for us. Therefore we can consider that the line that joins the two minima together is a good approximation for the bounce (as we already mentioned, this is an upper bound for the tension, as the bounce is the one that minimises the action). Then we can consider that $Y = \frac{\Delta Y^{(+,-)}}{\Delta X^{(+,-)}} X + b$ where by $\Delta X, Y^{(+,-)}$ we denote the difference in the value of X, Y between both minima, clearly $\Delta Y^{(+,-)} = \pi/a$, while the value of

[7] In units of $M_P = 1$.

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$\Delta X^{(+,-)}$ cannot be written analytically. If we use this relation between the fields X and Y we recover again the standard one-dimensional case where the canonical field will now be $\varphi^{(+,-)} = \sqrt{\frac{3}{2}(1 + (\frac{\Delta Y^{(+,-)}}{\Delta X^{(+,-)}})^2)} \ln X$.

In order for this model to be useful for explaining the actual acceleration stage of the universe and also the smallness of the cosmological constant we should check that the tunneling probability for the decay from $V_0^{(+)}$ to $V_0^{(-)}$ is larger than the tunneling probability for the decay from $V_0^{(+)}$ to the minimum at $V = 0$, and also that the minimum at $V_0^{(-)}$ has a decay time larger than the life of the universe.

Let us begin by computing the decay probability from the minimum at $V_0^{(+)}$ to the minimum at $V = 0$ and comparing it with the decay probability from the minimum at $V_0^{(+)}$ to the minimum at $V_0^{(-)}$. For those cases we will have from (6.4) that the probability will be given in both cases by

$$P \approx \exp\left(-\frac{24\pi^2/V_0^{(+)}}{(1 + 4V_0^{(+)}/3S_1^2)^2}\right), \tag{6.5}$$

as $V_0^{(+)} \gg V_0^{(-)}$. Therefore it is clear that the decay with smaller tension S_1 will be the most probable. The tension of the bubble wall in both cases can be written as

$$S_1^{(+,\infty)} \sim \sqrt{V_1^{(+,\infty)}}\,\Delta\varphi^{(+,\infty)}, \qquad S_1^{(+,-)} \lesssim \sqrt{V_1^{(+,-)}}\,\Delta\varphi^{(+,-)}. \tag{6.6}$$

where $V_1^{(+,\infty)}$, $V_1^{(+,-)}$ denote the height of the maxima that separates any two minima (see figure 4a and 4b). Also $\Delta\varphi^{(+,\infty)}$, $\Delta\varphi^{(+,-)}$ denote the variation of the canonical field in each case. As can be shown in figure 4 we have that $1 \gtrsim \Delta\varphi^{(+,\infty)} \gg \Delta\varphi^{(+,-)}$ and $V_1^{(+,\infty)} \gtrsim V_1^{(+,-)}$, so then we will have in general terms that $S_1^{(+,\infty)} > S_1^{(+,-)}$. Therefore we find that in this case it is more probable to decay to the minimum $V_0^{(-)} \sim 10^{-120}$ than to the minimum $V = 0$ at $X \to \infty$.

Now we must compute the probability of decay from the minimum at $V_0^{(-)}$ to the minimum at $V = 0$. In this case the tension of the bubble wall can be written as

$$S_1^{(-,\infty)} \sim \sqrt{V_1^{(-,\infty)}}\Delta\varphi^{(-,\infty)}, \tag{6.7}$$

where $V_1^{(-,\infty)}$ is the height of the maximum that separates the two minima. From the form of the potential shown in figure 4a, we can assume that $\Delta\varphi \sim \mathcal{O}(1)$, so it is clear from (6.7) that $S_1^2 \gg V_0^{(-)}$. Therefore for the decay probability one simply gets

$$P_{(-,\infty)} \approx \exp\left(-\frac{24\pi^2/V_0^{(-)}}{(1 + 4V_0^{(-)}/3S_1^2)^2}\right) \sim \exp\left(-\frac{24\pi^2}{V_0^{(-)}} + \frac{64\pi^2}{S_1^2} + \cdots\right). \tag{6.8}$$

For $V_0^{(-)} \sim 10^{-120}$ this probability is extremely small, so therefore, this dS vacuum can be considered stable in practical terms. Also note that the thin-wall approximation is always true within this model as $\mathcal{O}(1) \gtrsim \Delta\varphi$ and $\bar{\rho}, (V_0^{(\pm)})^{-1/2} \gg 1$.

It is interesting to note that the analysis of the stability of the last minimum in the $\mathcal{N} = 1^*$ is very similar to this last case. The reason for that is the following: the analytical discussion of the scalar potential in the non-supersymmetric case developed in section 4

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showed that apart from the minimum we also find a saddle point located in (for $D = D_0$)

$$X^0_{sp} = 1.12\sqrt{N} + 1.32 + 1.33\frac{1}{\sqrt{N}} + \cdots \tag{6.9}$$

$$Y^0_{sp} = 1.44\sqrt{N} + 0.37 - 1.36\frac{1}{\sqrt{N}} + \cdots \tag{6.10}$$

with a value of the potential given by

$$V^0_{sp} = 0.11\frac{1}{N^{3/2}} - 0.46\frac{1}{N^2} + 1.57\frac{1}{N^{5/2}} + \cdots \tag{6.11}$$

This value of the potential it is small for large N, but is still big compared with 10^{-120} for reasonable values of N. As this point is a maximum in one direction but a minimum in the other directions, we can perform an analysis of the stability of the vacua following the lines of the two exponential case. As in this case we also find $\Delta\varphi \sim \mathcal{O}(1)$ and $V_{sp} \gg V^0_{\min} \sim 10^{-120}$ we will arrive to the same conclusion as the one in (6.8).

In the $\mathcal{N} = 1^*$ potentials, as in all the cases with many minima, a typical situation may be that some of the minima would correspond to de Sitter space and others to anti de Sitter. We may imagine living in the one corresponding to de Sitter space with the smallest value of the cosmological constant and would wonder about its decay probability towards a global minimum with negative cosmological constant. As discussed in [21], the decay to a state of negative vacuum energy may or may not occurr and may lead to gravitational collapse. This has been recently reanalysed in [24].

Finally we would like to mention that it is also necessary to check that the decay times of the de Sitter vacua are also not too long. The fact that a de Sitter space has finite entropy introduces a time scale that is the Poincaré recurrence time t_r [25]. This quantity is given by $t_r \sim e^{S_{dS}}$, where S_{dS} denotes the entropy of the de Sitter space. For dS space the entropy has a simple sign-reversal relation with respect to the euclidean action calculated for the false vacuum dS solution $\varphi = \varphi_0$, which is given by $-24\pi^2/V_0$. Then the recurrence time can be written as $t_r \sim e^{24\pi^2/V_0}$. An interesting property of this kind of models is that the decay time of the de Sitter vacua never exceeds the recurrence time of the de Sitter space t_r. This was first noticed in [6] and can be easily checked from the following expression

$$\frac{\ln t_r}{\ln t_{\rm decay}} = \ln\left(1 + 4V_0^{(\pm)}/3S_1^2\right)^2 > 0 \quad \Longrightarrow \quad t_r > t_{\rm decay}\,. \tag{6.12}$$

The problems related to the decay time $t_{\rm decay}$ exceeding the recurrence time t_r will then not appear in these models.

7. Discussion

We have presented examples of multiple de Sitter vacua in string theory. Even though the examples we consider are still relatively simple, they illustrate what can be expected from the general vacuum structure of string theory, i.e. a multitude of vacua with different values of the cosmological constant.

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The parameters of the theory allow for one of the minima to have a cosmological constant as small as we want. This requires fine tuning[8] but it can be ameliorated given the large number of minima, indicating an anthropic approach to the cosmological constant problem, as advocated by different authors [10, 12]. The number of minima increases with the rank of the gauge group N. Furthermore since there is an underlying $\mathrm{SL}(2,\mathbb{Z})$ symmetry behind the $\mathcal{N}=1^*$ theory, we may expect that there could be further, possibly infinite, minima if we explore other fundamental domains of this group. We have essentially only explored N copies of the strips defined by $-1/2 < Y < 1/2$ for X outside the unit circle in the upper half plane. For each of these strips, the modular group has an infinite number of fundamental domains that could indicate a huge multiplication of the number of minima, inequivalent from the ones we found since $\mathrm{SL}(2,\mathbb{Z})$ is not a symmetry of the theory.[9] However their study is beyond the limit of validity of our effective actions which are trusted only for X greater than the string scale. Furthermore our potentials have periodicity N in the Y direction. Any correction that would slightly break this periodicity could give rise to an infinite number of minima.

The large number of vacua that can appear in these theories due to the nontrivial superpotential complements the already rich structure of vacua due to the presence of the fluxes [26]. The large number of 3-cycles in typical Calabi-Yau manifolds imply a large amount of possibilities. Remember also that although the combination of the fluxes KM is restricted by the tadpole cancellation condition, the ration K/M is a free (quantised) parameter.[10] This combination appears in the warp factor and allows the tuning of the parameter D to get a small cosmological constant, defining the 'discretuum' of vacua as described in [9, 12, 27]. All these effects were present in the single exponential case considered in KKLT. The large number of vacua we found has to be multiplied by this degeneracy. Although it may not be large enough degeneracy for a naturally small cosmological constant, the greater the number of minima the more natural is to find a cosmological constant of the right size.

The fact that our potentials depend nontrivially on at least two real fields X, Y makes the discussion of the system more interesting than for single field potentials in several ways. Since we may have many de Sitter minima, if we imagine the universe starting in any of them, it would leave naturally to different periods of inflation, either from tunneling between minima but also by naturally rolling after the tunnelings. There are so many valleys and hills in the potential that it may not be impossible to find regions of slow roll between different minima.

This combination of tunneling plus rolling has been considered in the past on different models of inflation such as open inflation [29, 30]. A detailed study of the possibility for

[8]The fact that we do not need to have a nonvanishing superpotential from the fluxes (W_0) indicates we do not need to fine tune this quantity as in [6]. However we still need to fine tune the supersymmetry breaking parameter D which even though is discrete it can be varied almost continuosly [6, 27].

[9]Notice also that in computing the scalar potential we restricted ourselves to one single phase of the $\mathcal{N}=1^*$ theory. In general we could consider any other values of p and q.

[10]Notice that typical four-folds can have Euler number between 10^3 and 10^6 [28], allowing for many different combinations of M and K to satisfy the tadpole condition.

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these potentials to give rise to realistic (eternal) inflation would be clearly of great interest. For this we recall that the main obstacle to have successfull inflation from string theory is precisely the lack of control of the moduli potentials. In particular the different proposals of D-brane inflation [31] assume that there is an unknown stringy mechanism that fixes the moduli and then, after that, D-brane inflation could occur. This is clearly a very strong assumption and so far attempts of combining D-brane inflation with moduli fixing have been running into problems.[11] Therefore we may consider seriously the possibility that actually the modulus T could be the inflaton field, and these potentials could give rise to interesting combinations of inflationary processes. This is clearly a possible subject for future investigation.

Another open question left unanswered here is the detailed analysis of the decays of the different minima. This is complicated by the facts that we have two-field potentials and that the fields do not have canonically normalised kinetic terms. Our discussion was mostly carried for the simplest case of two exponentials with only two minima, but clearly a complete analysis for the $\mathcal{N} = 1^*$ and other more general potentials would be needed.

Even though our models are relatively simple, they illustrate the potential richness of the string landscape once the different 'moduli' fields acquire nontrivial potentials. We do not pretend that the superpotentials discussed here, such as the one from the $\mathcal{N} = 1^*$ theory, would be particularly special over other more realistic realisations of non-perturbative potentials. Actually, we regard it as an interesting tool which includes all the ingredients expected from non-perturbative physics, in particular the infinite instanton sum can be under control thanks to the mathematical properties of modular forms. Other non-perturbative superpotentials recently derived, including different deformations of $\mathcal{N} = 4$ theory [18] would be interesting to study.

The mechanism of KKLT, although very interesting, takes the simplest class of models in which only one modulus is left unfixed by the fluxes. In more realistic models we would expect many moduli left unfixed by the fluxes and finding a many fields non-perturbative potential for them would be an important challenge. We regard our study here as a nontrivial, yet manageable, realisation of the multi-vacua scenario in string theory and hope it may be of interest to explore further properties of the space of string vacua.

8. Note added in revised version

The analysis presented in this article was performed using a supersymmetry breaking term induced in the potential due to the inclusion of an anti D3-brane in the configuration. Such a term was taken to be given by $\delta V = D/X^3$, with D a positive constant depending on the fluxes turned on in the compactification. Nevertheless, when this work was already finished it was pointed out in [32] that this induced term is not the one to be used if the anti D3 brane is in the warped region, as it is. The correct term should be $\delta V = \tilde{D}/X^2$ (with $\tilde{D}$ also a positive, flux-dependent, constant). This arises because the supersymmetry breaking term

[11] S. Kachru and J. Maldacena private communication [32].

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$N=50$			$N=100$			$N=100$		
X	Y	$V_{\rm min}$	X	Y	$V_{\rm min}$	X	Y	$V_{\rm min}$
1.70	10.98	$1.28\cdot 10^{-3}$	1.09	45.37	$3.79\cdot 10^{-2}$	1.94	18.12	0
1.97	18.96	$4.72\cdot 10^{-4}$	1.49	13.20	$9.10\cdot 10^{-3}$	2.06	27.47	$7.14\cdot 10^{-4}$
2.15	14.25	0	1.66	15.28	$2.13\cdot 10^{-3}$	2.35	22.29	$6.26\cdot 10^{-4}$
2.65	21.30	$9.33\cdot 10^{-4}$	1.80	38.15	$2.91\cdot 10^{-3}$	2.55	29.86	$1.02\cdot 10^{-3}$
			1.80	38.15	$2.91\cdot 10^{-3}$	2.70	36.87	$1.09\cdot 10^{-3}$
			1.82	41.84	$9.05\cdot 10^{-4}$			
$N=500$								
X	Y	$V_{\rm min}$	X	Y	$V_{\rm min}$	X	Y	$V_{\rm min}$
1.46	28.5	$1.26\cdot 10^{-2}$	2.13	108.6	$4.01\cdot 10^{-4}$	1.5	30.2	$8.04\cdot 10^{-3}$
2.20	113.7	$3.84\cdot 10^{-4}$	1.57	32.2	$4.34\cdot 10^{-3}$	2.23	47.6	$3.62\cdot 10^{-4}$
1.63	116.9	$3.46\cdot 10^{-3}$	2.23	59.7	$1.12\cdot 10^{-3}$	1.64	132.1	$7.10\cdot 10^{-3}$
2.25	65.4	$7.58\cdot 10^{-4}$	1.66	34.4	$2.14\cdot 10^{-3}$	2.48	68.1	$9.44\cdot 10^{-4}$
1.77	37.0	$5.37\cdot 10^{-4}$	2.51	52.7	$9.66\cdot 10^{-4}$	1.78	138.4	$2.14\cdot 10^{-3}$
2.53	146.2	$1.11\cdot 10^{-3}$	1.79	89.0	$1.97\cdot 10^{-3}$	2.58	74.8	$9.41\cdot 10^{-4}$
1.82	92.8	$1.14\cdot 10^{-3}$	2.60	139.5	$1.09\cdot 10^{-3}$	1.85	134.4	$5.09\cdot 10^{-4}$
2.68	139.4	$1.102\cdot 10^{-3}$	1.90	40.0	0	2.91	87.3	$1.15\cdot 10^{-3}$
2.04	58.0	$1.24\cdot 10^{-3}$	2.95	79.3	$1.11\cdot 10^{-3}$	2.05	43.5	$8.48\cdot 10^{-6}$

Table 7: Minima of the scalar potential for several non-supersymmetric cases.

coming from the inclusion of an anti D3-brane in the warped compactifications considered here scales like e^{4A}/X^3, and in the highly warped regime $e^{4A}\sim X e^{-8\pi K/3g_s M}$. This fact does not change the main conclusions of the paper although it changes the numerical analysis. We briefly summarize in this added note the changes produced in the numerical analysis.

As it was found in the D/X^3 case, with the introduction of the supersymmetry breaking term $\tilde{D}/X^2$ it is also possible to lift the vacua from anti de Sitter vacua to de Sitter vacua, for some range of the parameter $\tilde{D}$. Again we found that the effect of the supersymmetry breaking term depends on the range of values of the parameter $\tilde{D}$. If $\tilde{D}$ is very small (compared with the value of the potential in the supersymmetric case), the potential will not change substantially and the minima remain anti de Sitter. For a critical value of $\tilde{D}$ one of the minima will move up to zero vacuum energy and then to de Sitter space. Continuing increasing $\tilde{D}$, more minima become de Sitter until all of them are either de Sitter or Minkowski. We will denote by $\tilde{D}_0$ that precise value of the parameter $\tilde{D}$ such that all the values of the potential at the minima are positive or zero. If $\tilde{D}>\tilde{D}_0$ the nonsupersymmetric term starts dominating the potential and starts eliminating the different extrema to make the potential runaway with X.

Actually, we have computed numerically the minima of the potential for $\tilde{D}=\tilde{D}_0$ for several values of N, such as $N=50$, $N=100$ and $N=500$. The results obtained from the analysis are shown in table 7, where in those cases $\tilde{D}_0=0.04836,\ 0.04958$, and 0.04969 for $N=50,\ 100$, and 500 respectively.

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Also we show in figure 9 the scalar potential for $N = 100$ when the supersymmetry is broken. Note that comparing with figure 7 the main differences are that now the potential is less smooth and therefore has more minima (this can also be seen by comparing the information shown in table 5 and table 7).

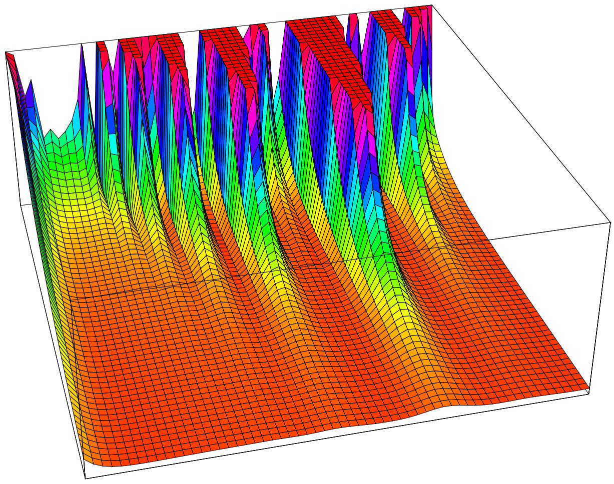

Figure 9: Graph for the non-supersymmetric scalar potential with $N = 100$.

In table 8 we show a similar information as the one shown in table 6, where we write the number of minima varying with N. In table 8 we denote by $\mathrm{Min}_{X,Y}$ the minima with largest value of X, Y and by $\mathrm{Min}_{V=0}$ the minima with vanishing value of the potential. We can notice that the number of minima increases with N more or less in the same rate than in the supersymmetric case and then faster than in the D/X^3 case. The reason for

N	Number Minima	Min_X X	Y	V_{min}	Min_Y X	Y	V_{min}	$\mathrm{Min}_{V=0}$ X	Y
10	1	2.4	3.31	0	2.4	3.31	0	2.4	3.31
20	2	3.41	4.83	$1.07 \cdot 10^{-3}$	3.41	4.83	$1.02 \cdot 10^{-3}$	1.95	7.84
30	2	2.33	12.13	0	2.33	12.13	0	2.33	12.13
40	4	2.54	16.74	$1.01 \cdot 10^{-3}$	2.54	16.71	$1.01 \cdot 10^{-3}$	1.94	11.32
50	4	2.65	21.32	$9.35 \cdot 10^{-4}$	2.65	21.31	$9.35 \cdot 10^{-4}$	2.15	14.25
60	5	2.91	25.88	$9.74 \cdot 10^{-4}$	2.91	25.88	$9.74 \cdot 10^{-4}$	1.84	13.23
70	6	3.26	30.52	$1.14 \cdot 10^{-3}$	3.26	30.52	$1.14 \cdot 10^{-3}$	1.98	15.53
80	8	3.62	35.09	$1.10 \cdot 10^{-3}$	3.62	35.09	$1.10 \cdot 10^{-3}$	2.10	17.74
90	8	2.61	33.38	$8.76 \cdot 10^{-4}$	1.73	37.73	$1.59 \cdot 10^{-3}$	1.82	16.28
100	10	2.7	36.87	$1.10 \cdot 10^{-3}$	1.82	41.84	$9.06 \cdot 10^{-4}$	1.94	18.12
200	21	2.84	53.81	$1.13 \cdot 10^{-3}$	2.84	53.85	$1.13 \cdot 10^{-3}$	1.88	87.82
300	23	2.84	123.86	$1.20 \cdot 10^{-3}$	2.26	124.10	$1.38 \cdot 10^{-3}$	1.94	31.54
400	22	3.15	75.72	$1.15 \cdot 10^{-3}$	2.54	83.10	$9.56 \cdot 10^{-4}$	1.88	87.71
500	27	2.84	123.81	$1.10 \cdot 10^{-3}$	2.26	124.10	$1.38 \cdot 10^{-3}$	1.94	31.50
600	28	3.34	95.55	$1.19 \cdot 10^{-3}$	2.81	167.72	$1.19 \cdot 10^{-3}$	1.92	44.41
700	29	3.43	76.41	$1.54 \cdot 10^{-3}$	2.27	148.15	$5.10 \cdot 10^{-4}$	2.05	55.16
800	29	3.43	76.41	$1.54 \cdot 10^{-3}$	3.21	109.92	$1.18 \cdot 10^{-3}$	2.05	55.16
900	29	3.46	133.21	$1.14 \cdot 10^{-3}$	3.38	133.21	$1.18 \cdot 10^{-3}$	2.04	58.05
1000	32	3.15	120.73	$1.19 \cdot 10^{-3}$	3.07	129.22	$1.19 \cdot 10^{-3}$	2.02	60.59

Table 8: Minima for different values of N in a non-supersymmetric case with $\tilde{D} = \tilde{D}_0$.

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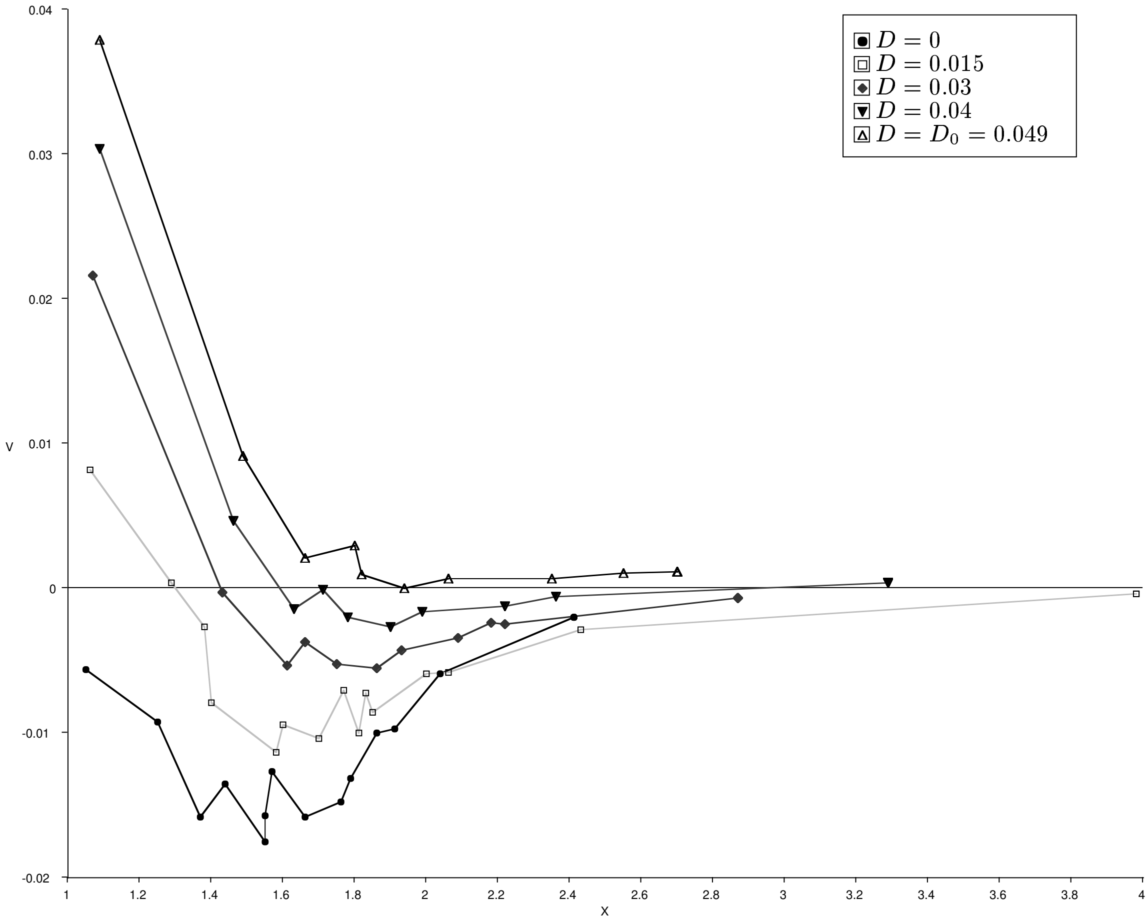

Figure 10: Potential versus X for $N = 100$ and different values of $\tilde{D}$.

this is that the value of $\tilde{D}$ for which all the anti de Sitter minima of the potential get lifted is smaller than in the D/X^3 case, and therefore in the present case the δV term smooths less the potential.

Also, in figure 10 we illustrate the effect of the corrected non-supersymmetric term in the potential. The value of the potential at the minima is presented for several values of the parameter $\tilde{D}$. For $\tilde{D} = 0$ we have the supersymmetric case, the increasing of the parameter $\tilde{D}$ will reduce the number of minima and increase the value of the compactification scale at the minima. Note that now the minima are not ordered for increasing X, and also that the values of the compactification scale at the minima are, in general, smaller than in the D/X^3 case but still large enough so that our approximations remain valid.

Summarising: the difference with the $1/X^3$ potentials is that now there are more de Sitter minima, they are no longer ordered with respect to the size of the extra dimension and the value of the volume at the minima tends to be smaller than in the $1/X^3$ case, but still large enough for the large radius approximation to be trustable.

Finally we would like to point out that the $1/X^3$ behaviour of the nonsupersymmetric part of the potential presented in the main text is still relevant. This is due to the recent proposal in [33] in which instead of introducing an anti D3 brane on the throat, a flux of magnetic fields on the D7 branes is considered. This adds an extra term in the potential which happens to be proportional to $1/X^3$ if the D7 brane is not sitting on the throat and to $1/X^2$ if the D7 brane is on the throat. In each case the rest of the analysis is as presented in here and therefore both potentials are relevant depending on the location of the D7 branes.

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